NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 07BAMS	**LEVEL:** 6
COURSE CODE: LIA601S	**COURSE NAME:** LINEAR ALGEBRA 2
SESSION: JUNE 2019	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS:** 100

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	Dr S.N. NEOSSI NGUETCHUE AND Pr A. KAMUPINGENE
MODERATOR:	Mr B. OBABUEKI

INSTRUCTIONS

1. Answer ALL the questions in the booklet provided.
2. Show clearly all the steps used in the calculations. All numerical results must be given using 3 decimals where necessary unless mentioned otherwise.
3. All written work must be done in blue or black ink and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 2 PAGES (Including this front page)

Attachments

None

QUESTION 1 [50 Marks]

1.1. Suppose that we know for a linear transformation of $\mathbb{R}^2$ that $T((1,1)) = (3,5)$ and $T((-1,2)) = (0,1)$.

1.1.1 Find the matrix A such that $T(x) = Ax$. [8]

1.1.2 Given the basis $\mathcal{B} = \{(1,-2),(3,3)\}$, find the matrix B so that $T[x]_{\mathcal{B}} = B[x]_{\mathcal{B}}$ (that is A and B are similar relative to the basis $\mathcal{B}$.) [9]

1.1.3 Find the $\mathcal{B}$-coordinates of the vector $x = (2,5)$ using the basis in **1.1.2** above. [7]

1.2. Let $T: P_2(\mathbb{R}) \to P_2(\mathbb{R})$ be a mapping defined by

$$T(f(x)) = f(x) + (1+x)f'(x), \text{ for any } x \in \mathbb{R}, \text{ where}$$

$P_2(\mathbb{R})$ is the set of all polynomials of degree at most 2 with real coefficients.

1.2.1 Show that T is a linear operator. [10]

1.2.2 Find all the eigenvalues of T. (Hint: use the basis $p_1 = 1, p_2 = x, p_3 = x^2$) [10]

1.2.3 Find all the eigenvalues of the operator $L = T^5 + 2T^3 + 5T$. [6]

QUESTION 2 [20 Marks]

Consider the matrix $P = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 3 & 1 \end{pmatrix}$.

2.1. Find a diagonal matrix D similar to P. [17]

2.2. Deduce from the previous question the computation of P^5. [3]

QUESTION 3 [30 Marks]

3.1. Let $A \in \mathcal{M}_{n \times n}(\mathbb{R})$ satisfy $A^3 = A$. Show that A is diagonalizable. [7]

3.2. Let A be a 4×4 matrix defined by

$$A = \begin{pmatrix} 2 & 0 & 1 & -3 \\ 0 & 2 & 4 & 8 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}$$

3.2.1 Find the minimal polynomial of A. [16]

3.2.2 Find a Jordan canonical form J of A. [7]

END OF PAPER
TOTAL MARKS: 100

God bless you !!!